JOINT INSTITUTE
FOR NUCLEAR RESEARCH

FRANK LABORATORY
OF NEUTRON PHYSICS

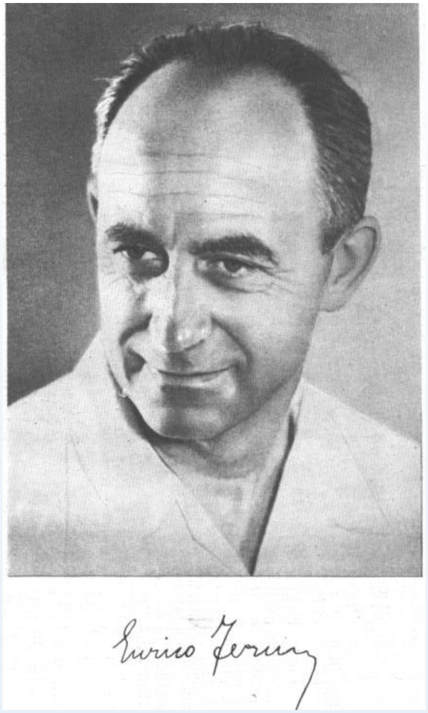
Direct measurement of the neutron velocity in a refractive medium and test of the dispersion law for UCN

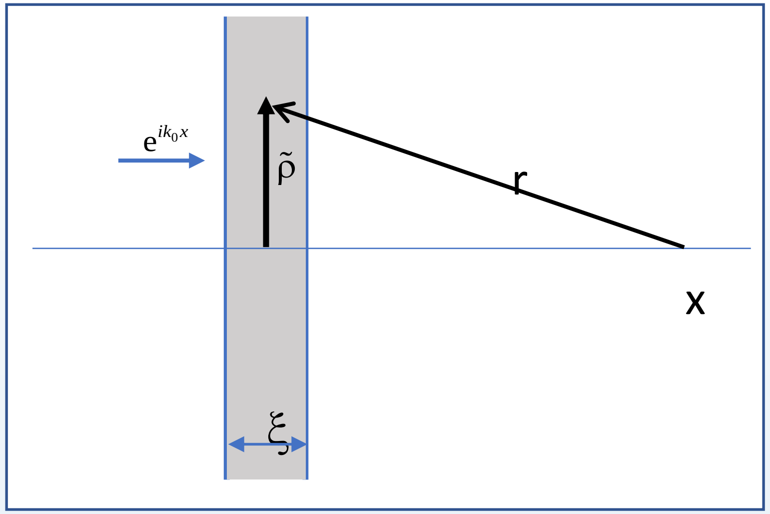
Alexander Frank

frank@jinr.ru

ISINN 30, 2024

Refraction index and dispersion relation for the neutron waves





E. Fermi, 1944-1950

$$k^2 = k_0^2 - 4\pi\rho b$$

L.Foldy,1945

Potential-like dispersion law

$$U = \frac{2\pi\hbar^2}{m} Nb$$

Effective potential (never was used by E. Fermi)

$$n^2 = 1 - \frac{4\pi}{k_0^2}\rho b$$

$$b = b' - ib''$$

Small corrections to the dispersion law of neutron waves 1.

$$k_1^2 = k_0^2 + 4\pi\rho C f_0$$

$$C = \begin{cases} C = \dfrac{1}{1-(4\pi/3)\rho\alpha} & \text{for light} \\ C = 1 \; (f_0 = -b) & \text{for neutrons} \end{cases}$$

Lax, 1951

I. M. Frank's hypothesis: for neutrons also $C \neq 1$

The presence of a very small imaginary part of the coefficient *C* leads to a noticeable change in the absorption coefficient

$$n^2 = 1 - \frac{4\pi\rho}{k_0^2}(C' - iC'')(b' - ib'')$$

$b''/b' \approx 10^{-4}\text{-}10^{-5}$

$C''b' \cong b''$

I.M. Frank, 1974

Corrections to the dispersion law of neutron waves.

$$n^2 = 1 + \frac{4\pi}{k^2}\rho f C, \qquad f = -b + ikb^2 \qquad C = (1-J)^{-1} \approx 1 + J' + J''$$

$$C' \approx 1 + 2\pi\rho b' a^2 \qquad \boldsymbol{ka} \to 0$$

$$J = Nb\int \exp(i\mathbf{k}\cdot\mathbf{r})G(\mathbf{r})\left[1-\mathbf{g}(\mathbf{r})\right]d\mathbf{r} \qquad G(\mathrm{r}) = \exp(ikr)/r$$

$$C'' \approx \pi\rho b' k a^3$$

a – *interatomic distance*

For UCN

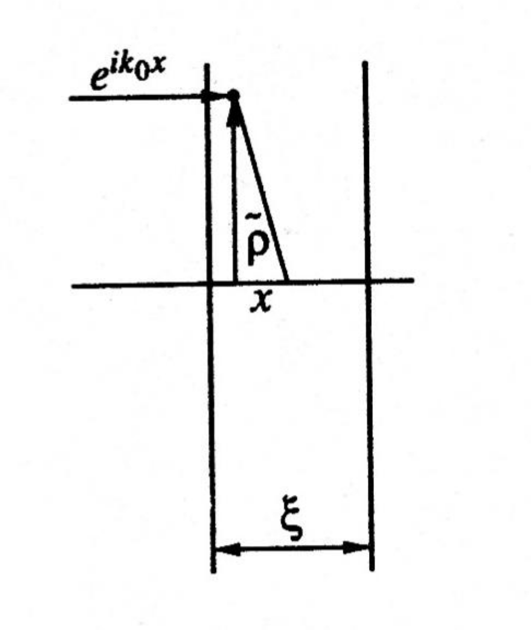
$C'\text{-}1 \approx 5\times10^{-4}$ (Almost constant!) $\quad C'' \approx 10^{-5} (\propto k)$

V.F. Sears,1982

$$n^2 = 1 - \frac{4\pi\rho b/k_0^2}{1 + \left(4\pi\rho b / nk_0^2\right)\int e^{ik_0 x}\sin(nx)\left[g(x/k_0) - 1\right]dx}$$

M. Warner & J.E.Gubernatis,1985

On the region of applicability of the potential-like dispersion law and hypothesis of super slow neutrons.



$$\psi_{scat} = 2\pi\rho b \frac{e^{ik_0x}}{ik_0}\xi$$

$$|\psi_{scat}| << 1, \quad \xi \gg a \approx \rho^{-1/3} \quad \frac{2\pi\rho b}{k_0} a << 1$$

If $k_0 \le 4\pi\rho b a$ then there is re-scattering at distances $\hat{\rho} = \left(k_b^2 a\right)^{-1}$

That leads to uncertainty $\Delta k \approx \hat{\rho}^{-1} \approx k_0$

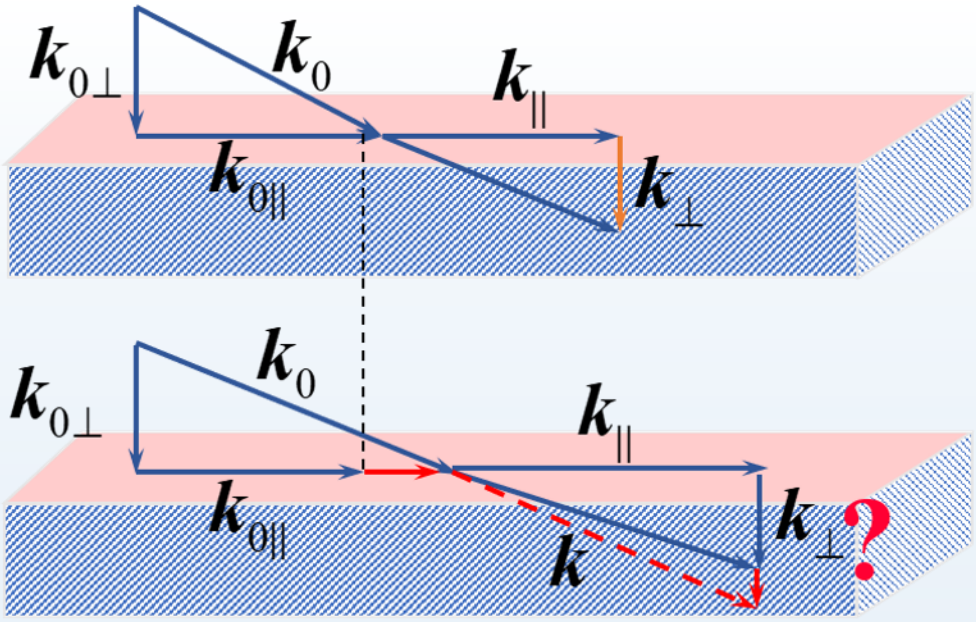
Region of applicability of the potential-like dispersion law $k_0 >> 4\pi\rho b a = \chi^2 a$ $\chi^2 = 4\pi\rho b$

V.G.Nosov & A.I.Frank, 1991

Dispersion law for super-slow neutrons $(v \le 10\mathrm{cm/s})$ strictly speaking unknown.

For UCN small corrections for the potential like dispersion law are very probable

Specific feature of the potential-like dispersion law



$$k_{II}^2 = k_{0II}^2$$

$$k^2 = k_0^2 - \chi^2; \qquad \chi^2 = 4\pi\rho b$$

$$k_\perp^2 = k_{0\perp}^2 - \chi^2; \qquad b = const$$

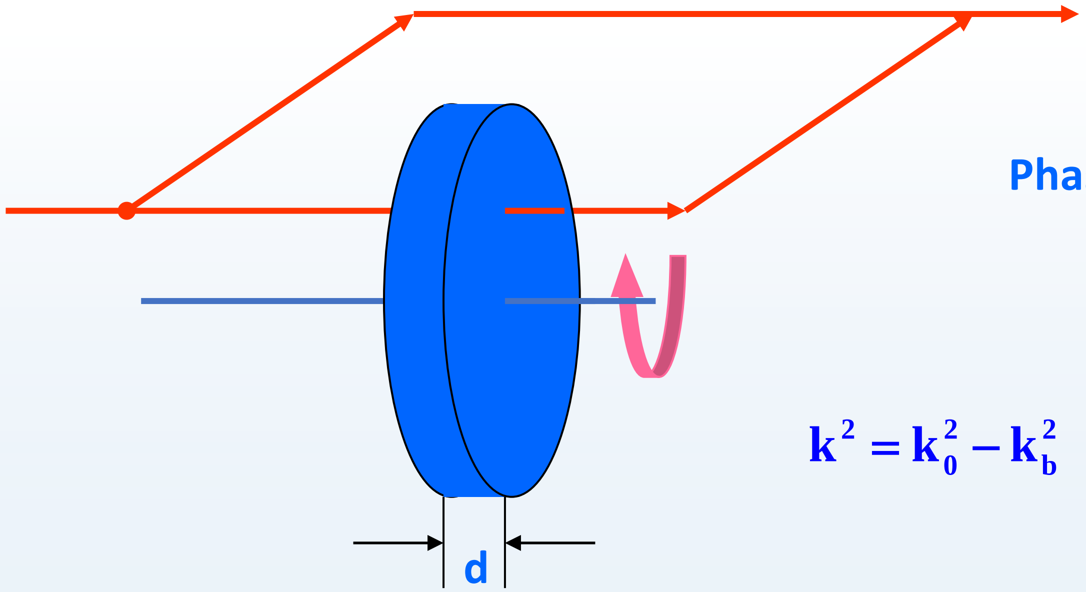
I.M.Frank, 1974,
A,G.Klein, S.A.Werner, 1983

$$k^2 = k_0^2 - \chi^2 + \varepsilon\left(k_0^2\right); \qquad \chi^2 = 4\pi\rho b$$

$$k_\perp^2 = k_{0\perp}^2 - \chi^2 + \varepsilon\left(k_0^2\right);$$

Strategy of the test experiment – looking for $k_\perp$ with variation of $k_{||}$ at $k_{0\perp} = \mathrm{const}$

Fizeau-type experiment with UCN



Phase shift $$\varphi = k(1-n)d$$

$$\mathbf{k}^2 = \mathbf{k}_0^2 - \mathbf{k}_b^2 \qquad \mathbf{k}^2 = \mathbf{k}_0^2 - \left[\mathbf{k}_b^2 + \varepsilon(\mathbf{k}_0^2)\right]$$

$$n^2 = 1 - \frac{k_b^2}{k_0^2} \qquad n^2 = 1 - \frac{\left[k_{lv}^2 + \varepsilon\left(k_0^2\right)\right]}{k_0^2}$$

Experiments for the test the validity of the potential-like dispersion law of the thermal neutrons

Thermal neutrons

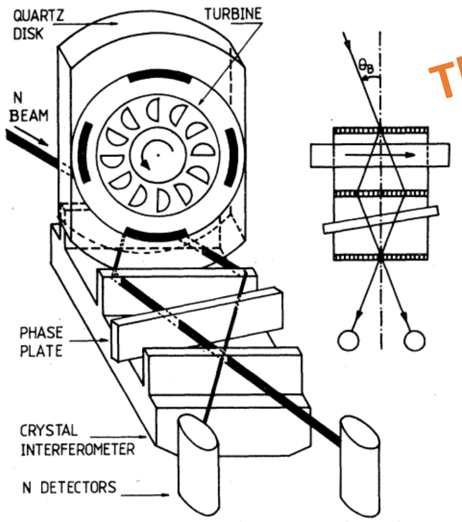


FIG. 1. Overall layout of neutron interferometer showing rotating quartz disk. Inset: schematic of neutron interferometer.

M. Arif, H. Kaiser, S.A.Werner et al.
Phys. Rev.A 31 (1985) 1203

The phase shift caused by rotation was not observed due to insufficient sensitivity of the experiment

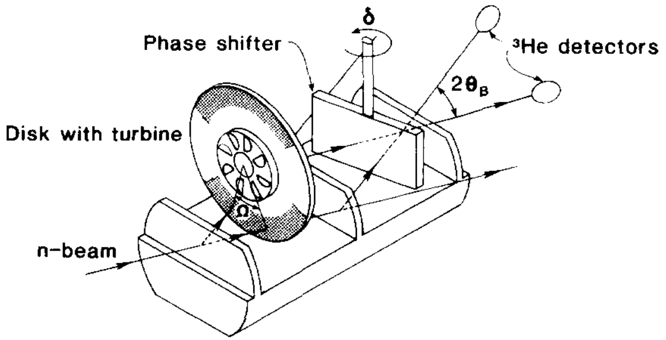


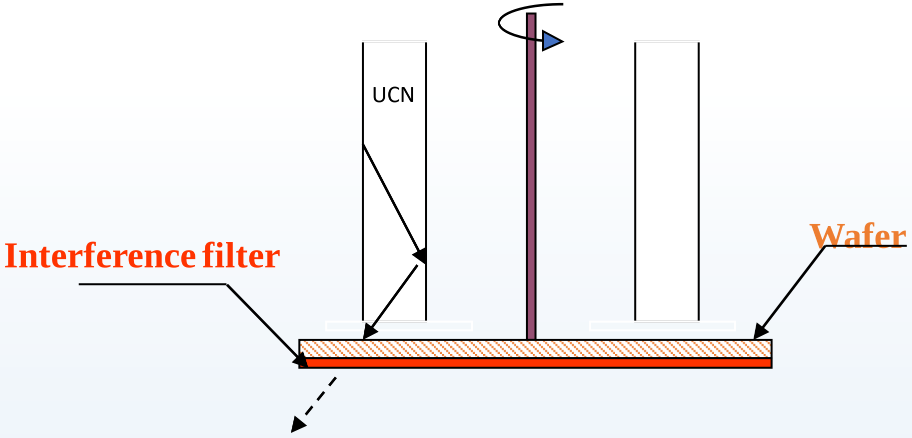
Fig. 2. Overall layout of the neutron interferometer showing the placement of the aluminum disk. The shaded segments on the disk correspond to the positions of the samarium foils.

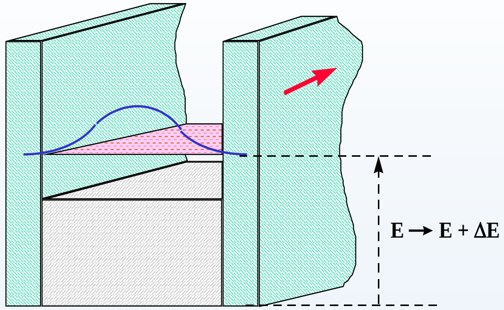
M. Arif, H. Kaiser, R. Clothier et al.
Physica B 151 (1988) 63-67

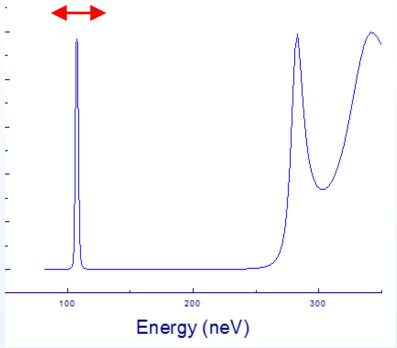
Prove of the principle. Observation of the effect for the matter with resonant cross-section $b \neq \text{const}$

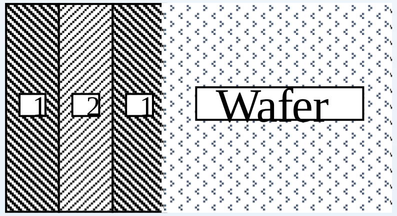
There is no now neutron interferometers for either UCN or VCN although such proposals have been made.

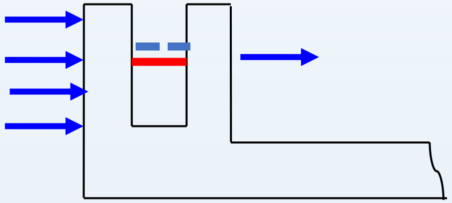
Rotating interference filter and experimental test of the dispersion law for UCN











$$U_{1,2} = \frac{2\pi\hbar^2}{m}(\rho b)_{1,2}$$

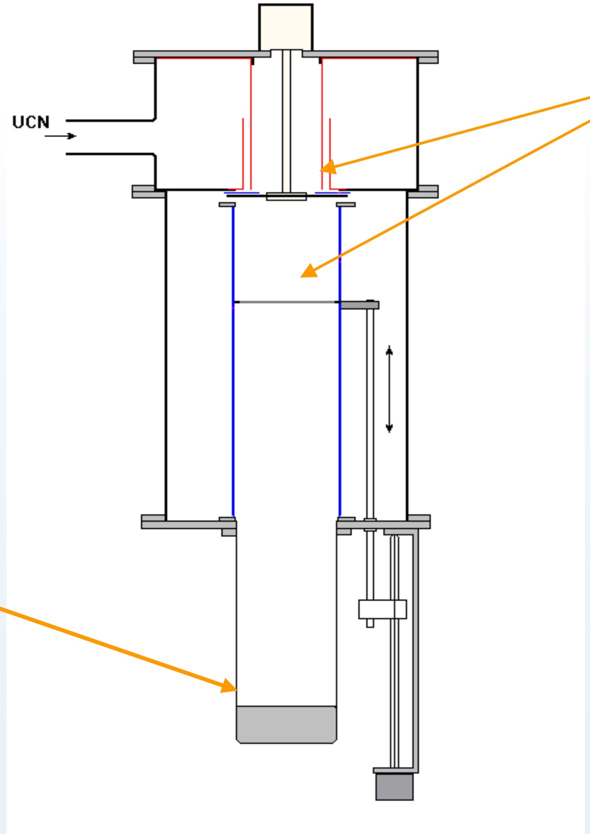
In case of deviation $(\varepsilon \neq 0)$ from the potential-like dispersion law the position of resonant should shift when the filter is spinning

$$k^2 = k_0^2 - 4\pi N b + \varepsilon\left(k_0^2\right)$$

$$k_\perp^2 = k_{0\perp}^2 - 4\pi N b + \varepsilon\left(k_0^2\right)$$

V.G.Nosov and A.I.Frank. Phys. At.Nucl. 58, 402 (1995) ISINN 3 (1995)

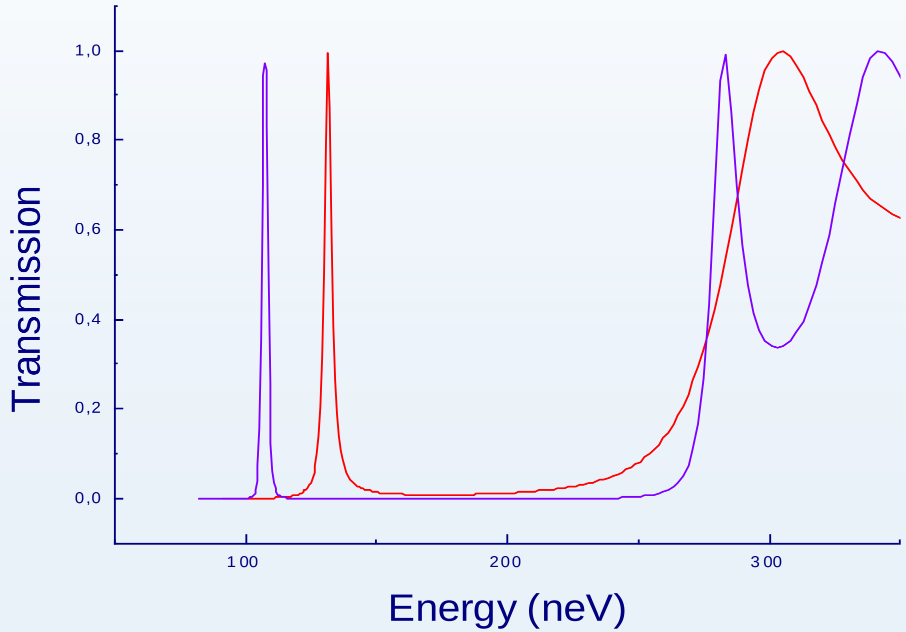
Gravity UCN spectrometer with Neutron Interference filters



Two NIF with variable space between them

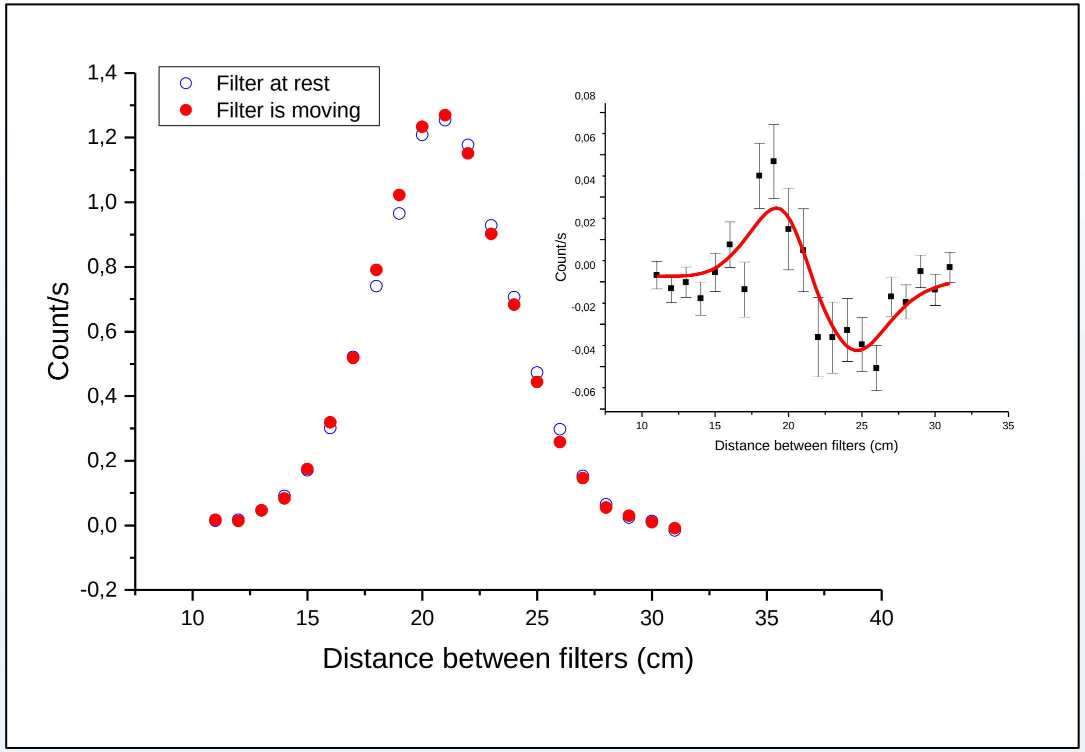
Detector

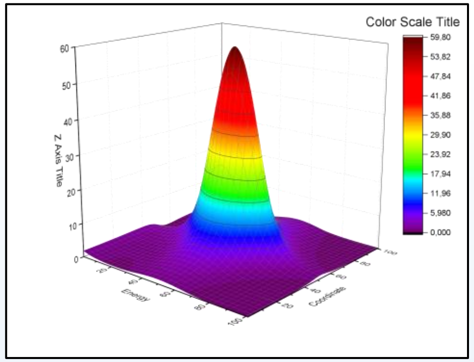
mg=1.02 neV

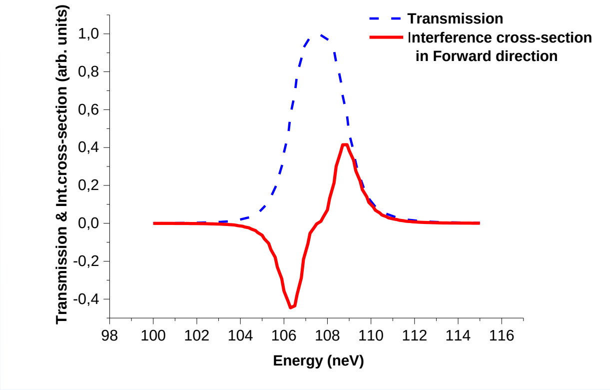


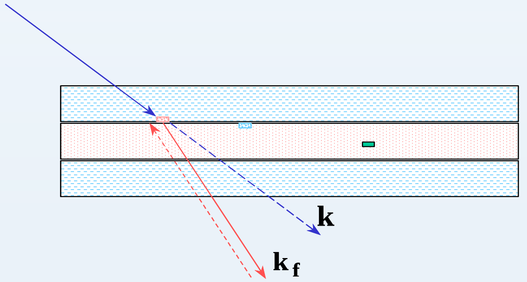
I.V. Bondarenko, A. V. Krasnoperov, A. I. Frank at al.
ISINN 7 (1997)

Resonant tunneling of UCN. New effect as a source of methodical error









$$f\left(k',k\right) = -\frac{m}{2\pi\hbar^2}\int \tilde{\Psi}_f\left(\vec{r}\right)V_1\left(\vec{r}\right)\Psi\left(\vec{r}\right)d\vec{r}$$

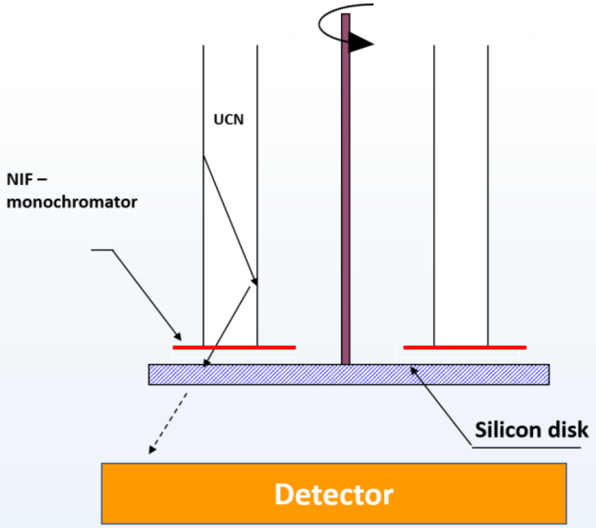
$$\sigma_{int} = -\frac{4\pi}{k} Im\left\{T^* f\left(k_t,k_t\right)\right\}$$

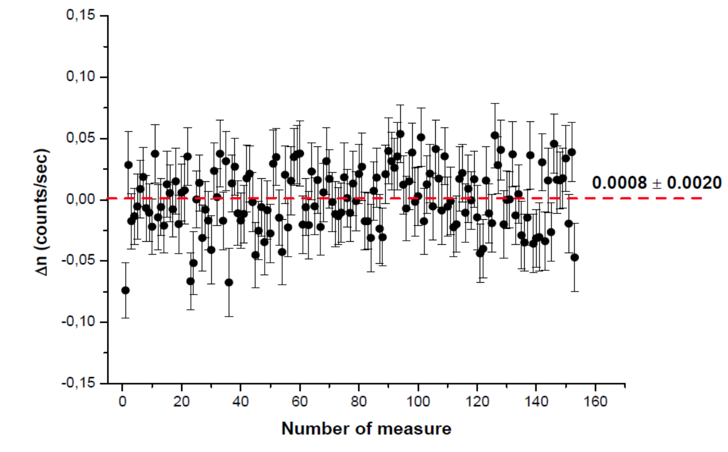
Previously unknown features of resonant tunneling of neutrons have been found which prevents to use NIF for the test of the dispersion law for UCN

I.V. Bondarenko, A. V. Krasnoperov, A. I. Frank at al.
ISINN 7 (1997)
JETP Letters, 67, (1998) 786.

A.I.Frank, S.N. Balashov, I.V. Bondarenko et al. 2001

New experiment for the test of the validity of the potential-like dispersion law for UCN





$$v_x = 6 \Leftrightarrow 36\ m/s$$

$$\Delta n / n = (0.6 \pm 1.4) \times 10^{-3}$$

Transmittivity depends on two parameters: real and imaginary parts of “potential”

$$U = V - iW = \frac{2\pi\hbar^2}{m} N (1 + J' + iJ'')(b' - ib'')$$

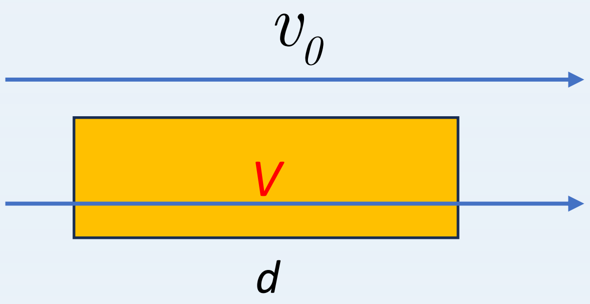
$$\delta J' = \ \le 3\times10^{-3} \ if \ \ \delta W = 0$$

$$\delta J'' = \ \le 3\times10^{-8} \ \text{if} \ \ \delta V = 0$$

G.V. Kulin et al, 2014

To verify the validity of the potential-like dispersion law for UCN we propose to measure the time delay of a neutron passing through the refractive sample.

In such experiment will be measured velocity but not a wave number.

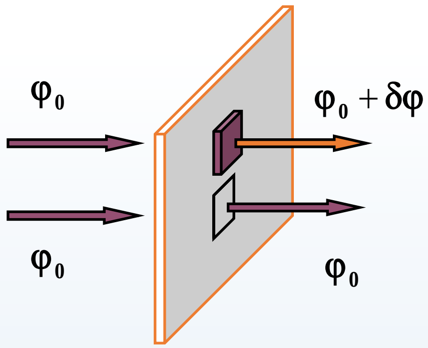


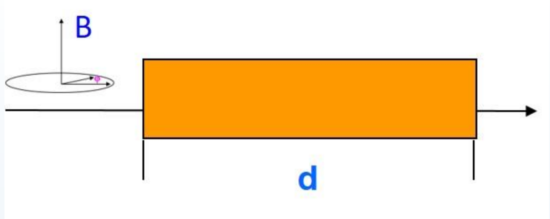
$$\Delta t = \frac{d}{V} - \frac{d}{v_0}$$

$$\Delta t = \frac{d}{v_0}\left(\frac{1-n}{n}\right)$$

$$V = n v_0 \ ?$$

What do we know from the experiments concerning neutron velocity inside the matter? Cold neutrons

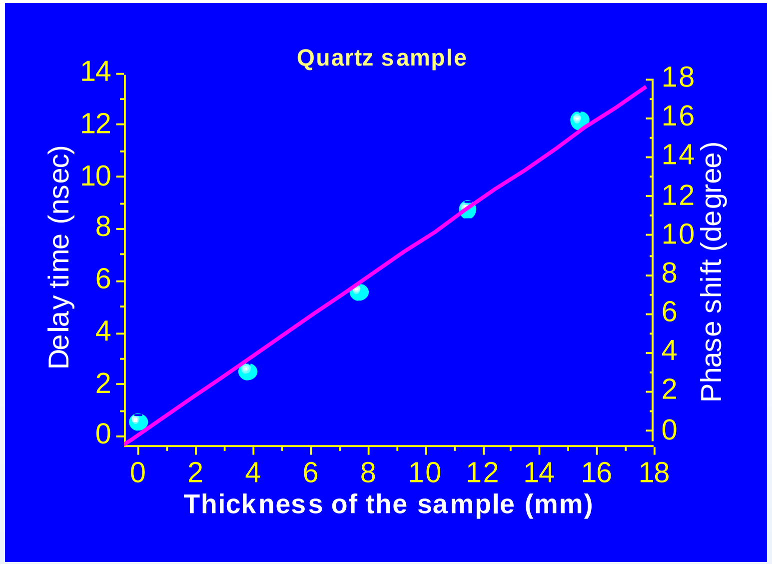




$$\Delta\Phi = \omega_L \left(\frac{1-n}{n} \right) \frac{d}{v_0}$$

Larmor clock

A.Frank et al. 2001



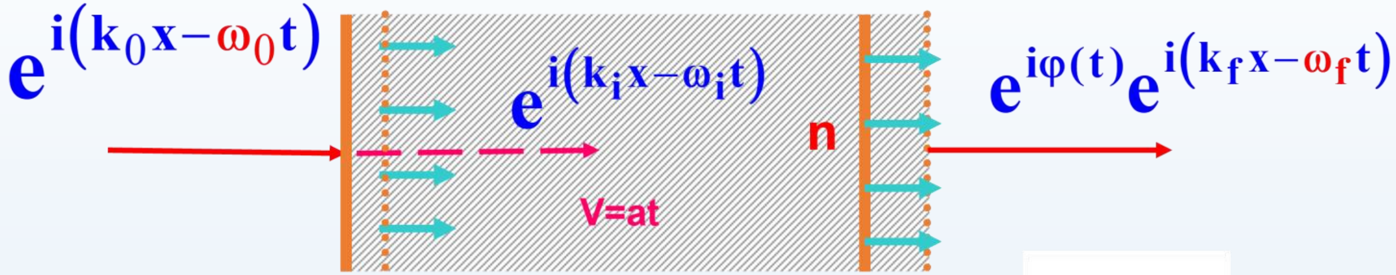
Material	ρb, 10^{-8} Å^{-2}	
	experiment	tabular value
Si	2.09 ± 0.03	2.15
Be	9.65 ± 0.02	9.63
Graphite	7.21 ± 0.13	7.5

The precision $\delta(\rho b)/\rho b \approx 0.005$

$V = nv_0!$

What do we know from the experiments concerning neutron velocity inside the matter? UCN

Accelerating Medium Effect



$$\omega_f \neq \omega_0$$

$$\Delta\omega = ka\Delta t \qquad \Delta t = \frac{d}{v_0}\left(\frac{1-n}{n}\right)$$

$$\Delta E = \hbar\omega$$

The aim of the experiments was to detect firstly new and rather small optical effect

The results were in reasonable agreement with theoretical predictions based on several assumptions.

Declared precision was about **7÷10%** $V = nv_0!$

A.I. Frank, P.Geltenbort, G.V.Kulin, et al, Phys. At. Nuclei, 71 1656 (2008) .
A. I. Frank, P.Geltenbort, M. Jentschel, et al.. JETP Letters, 93 361, (2011)

What do we know from the theory concerning neutron velocity inside the matter?

A.Frank, 2019

In general case dispersion law may be represent as $k = F(k_0^2)$

It was shown recently that in this case effective mass of neutron in a matter is $m^* = 2mkF'$

Due to k-number in a matter is $k = nk_0$ *neutron velocity in a matter is* $V = \frac{\hbar k}{m^*}$ $V \neq nv_0$ *if* $\varepsilon \neq 0$

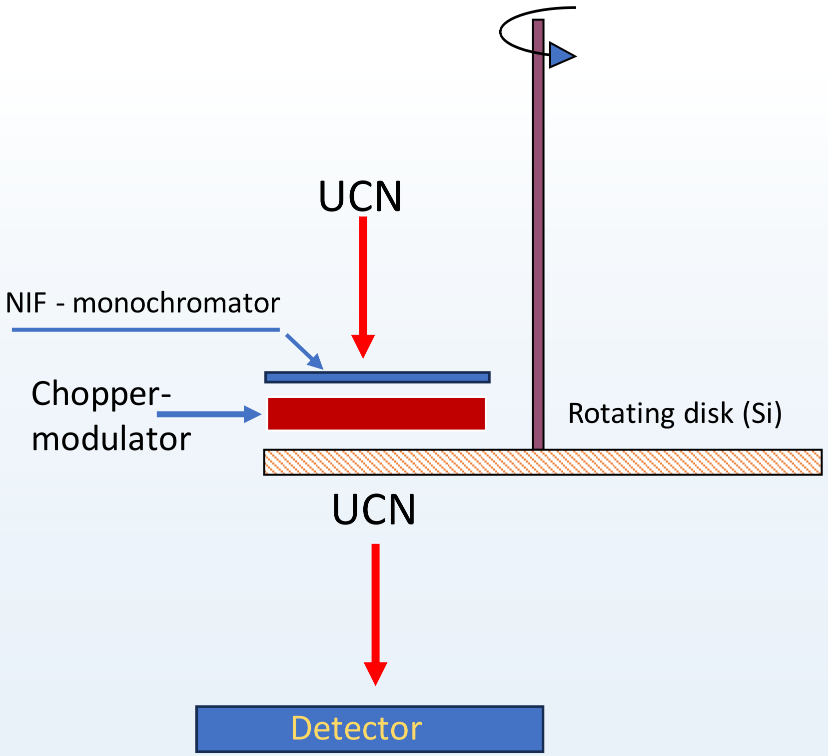
If we suppose that $k^2 = k_0^2 - 4\pi\rho b + \varepsilon\left(k_0^2\right)$ *then the flight time of a sample with a thickness of d is* $\tau = \frac{m^* d}{\hbar k}$

$$\tau = \frac{mL}{\hbar n k_0}\left(1 + \frac{d\varepsilon}{dk_0^2}\right)$$

$$n = \left[1 - \frac{4\pi\rho b - \varepsilon\left(k^2\right)}{k_0^2}\right]^{1/2} \qquad m^* = m\left(1 + \frac{d\varepsilon}{dk_0^2}\right)$$

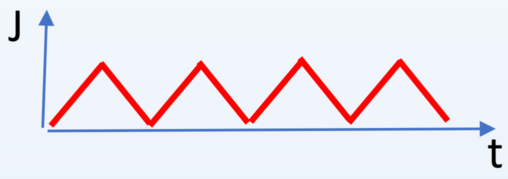
The experiment for the measure the neutron flight time through refractive sample is sensitive to the correction term to potential like dispersion law !!

Proposed experiment

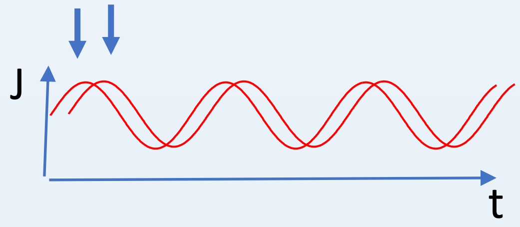


$$k^2 = k_0^2 - 4\pi\rho b + \varepsilon\left(k^2\right) \quad ??$$

$$\tau = \frac{md}{\hbar n k_0}\left(1 + \frac{d\varepsilon}{dk_0^2}\right)$$



$$\Delta t = \Delta\Phi/\omega$$



The aim of the experiment is to compare phase shifts of the count rate oscillation for the case when sample is at rest and is spinning

UCN 110 neV. The estimated delay time due to refraction in silicon with thickness of 2mm is about 200 mks

I am very grateful to my colleagues

German Kulin and Maxim Zakharov

in close collaboration with whom this work was done

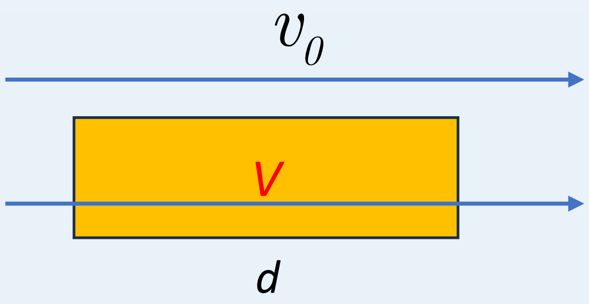
Thank you for you attention!

Outline

- **Introduction. Well known relations concerning refractive index and dispersion law for neutron waves**
- **Theoretical predictions for the corrections to dispersion law of cold neutrons**
- **Specific properties of the potential-like dispersion Law (PLDL)**
- **Experimental approaches to the test of the PLDL validity**
- **Proposal of the alternative approach**

To verify the validity of the potential-like dispersion law for UCN we propose to measure the time delay of a neutron passing through the refractive sample.

The main motivation is that such time delay depends only on difference of neutron velocities in vacuum and inside the matter. And velocity unlike the wave number, is a real value.



$$\Delta t = \frac{d}{V} - \frac{d}{v_0}$$

$$\Delta t = \frac{d}{v_0}\left(\frac{1-n}{n}\right)$$

$$V = n v_0 \ ?$$